478 Nuclear Theory ([illegible])

SPIRAL

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United States Patents
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Other Patents Pending

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Physics 478 - Nuclear Theory

Physics 278 – Nuclear Theory – Prof. Bethe – 2nd Term – 1939-40

Feb. 14

References: Bethe, Rev. Mod. Phys. '36, '37

Stationary States of Nuclei

Theory of Nuclear Reactions

Experimental Results.

Rasetti, Nuclear Physics – '36

Rev. Mod. Phys., '40 Artificial Radioactivity.

Particles occurring in nuclei : Protons, Neutrons.

" not " " " : + & – electrons, mesons of any sort.

For these reasons :

1) If a particle is in a system, collision with another system may eject it ; e.g. atomic collisions eject electrons, which are constituents of atoms – Particles ejected from nuclei are always heavy particles (during collisions – electrons only come out afterwards (radioactivity just as light is emitted from atoms (except that when electron is emitted, charge of nucleus changes) .

2) Nuclei are very small, electrons very light; By uncertainty principle, electron in space Δx has uncertainty $\gtrsim \frac{\hbar}{\Delta x}$; (relativistically) $E \sim cp \sim \frac{\hbar c}{\Delta x} \sim$ (e.g.) $\frac{10^{-27} \cdot 3\times 10^{10}}{10^{-12}} \sim 3 \cdot 10^{-5}$ erg

~ 20 mev.

Which is too much; would require a potential barrier of about 10^9 ev to keep electron in nucleus; such large ones don't exist —

3) Statistical argument —

electrons obey Fermi statistics: $\psi(r_1 r_2 \cdots) = -\psi(r_2 r_1 \cdots)$

r_1 - spin & space coord of 1st electron.

Pauli principle follows; no 2 electrons with same $n\, l\, m_s\, m_l$ — hence periodic table

Protons, also Fermi stat., spin ½

Deuterons, Bose stat., spin 1

$\psi(r_1, r_2 \cdots) = \psi(r_2 r_1 \cdots)$

If deuteron is composed of p & el., must be: $D = 2P + e$;

But: suppose we have systems A, B, each with n particles, where jA, jB are like particles; if they obey Fermi stat., then:

$$\psi(r_{1A} r_{2A} \cdots r_{nA}; r_{1B}, r_{2B} \cdots r_{nB}) = (-1)^n (r_{1B} r_{2B} \cdots r_{nB}; r_{1A} r_{2A} \cdots r_{nA})$$

But that is equivalent to interchanging the systems;

Hence if n is even, system should obey Bose,
" n " odd, " " " Fermi Stat.

if $D = 2P + e$, n is odd, & D should have Fermi stat, but it doesn't — in fact it obeys Bose; Hence $D = P + N$

n even — all OK.

Feb. 16, 1940 even A, odd z : A - mass number.

Z - charge.

Bose Statistics : D, Li^6 B^{10} N^{14}

Proton-electron hypothesis : A protons

A - Z electrons.

total 2A - Z.

Hence nuclei of even Z should obey Bose,

odd Z - - - - Fermi statistics ;

but D, N^{14}, which have odd z, have Bose stat.;

Proton-Neutron hypothesis :

Z protons

A - Z neutrons

A particles

even A - Bose — observed for D, N^{14}.

Odd A - Fermi — observed for C^{13}.

Nuclear charge Z is found from chemical properties ;

or from frequency of x-ray lines

(esp. K_α line : $2p \to 1s$ $\frac{3}{4} R_y Z^2$ + Re Co

Mass Number A : nearest integer to atomic wt. (found from mass spectrograph with accuracy of 1 in 10^5 for lighter, less for higher

Problem of Nuclear Theory:

1) Is description by protons + neutrons sufficient?

e.g. can magnetic moment of nucleus be found by (q-mechanical) vector addition of mm of constituents?

If not, protons + neutrons behave differently from free pr. & neutr.; and we must find theory to account for different properties, bound + free. (In atom, electrons behave the same whether bound or not)

Not much evidence on this. Probably pr-neutr. picture is good within a few percent.

2) What are forces between protons + neutrons? (Most fundamental). Nuclear forces are not electrical, nor gravitational — nothing, in fact, known from macroscopic experiments — In atom, forces were known — but how to describe motion wasn't found till 1926 — Q Mech. In Nucleus, we know (maybe) the rules of motion — but not forces

3) Is Q. Mech. adequate to describe motion of particles in nucleus?

Bethe thinks likely yes. No evidence to contrary.

Will proceed on assumption ~~that~~ it is.

If forces & mechanics are known, it is only a mathematical problem to solve problem for eigenvalues, eigenf'ns etc. — Very complicated, however; in atom Hartree approximation can be used; but not in nucleus — a good approx. not found yet.

This not a fundamental question; merely one of calculation.

Such Questions:

1) Which approximations can be used in calculation?

No good ones yet — other methods have been used — statistical theory of properties of various levels have been used —

2) Theory of compound Nucleus

Theory of reactions involving complicated nuclei.

Possible to predict accurately prob. of nuclear reaction — in fact, actual cross-sections — with no assumption about forces except ~~that they~~ are large —

Good for predicting exp. results —

Bad for finding real nuclear forces —

Only hope for finding forces, is to investigate simple nuclei. — Simplest nuclei are D (1P, 1N)

Triton T

He^{III}, He^{4} (alpha particle)

But if attempt is made to study $A > 4$, difficulties involved are tremendous, & results don't depend sensitively on forces involved –

Other problems:

Relations between heavy & lighter particles –

1) Mechanism of β-disintegration

To conserve energy, assume neutrino emitted –

Mechanism of theory by Fermi –

Must have a field theory for forces; that's another problem:

2) Field theory of nuclear forces

Properties of proton & neutron — intimately connected with field – (just as properties of electron are closely connected with electromagnetic field).

Field subject to quantization (e.g. photons, for e.m. field)

" of nuclear forces may also give rise to particles –

Greatest progress so far in Theory of comp. nucleus.

Some progress in question of nuclear forces — know approx magnitude & range –

Mechanism of β-disintegration – at least preliminary theory

Pr.-Neut. structure, & applicability of Q.M. – yes, close enough to assume it. Field theory only in beginning stages –

Likewise question of usable approximations — much work done, but not much results.

2/21/40) Nuclear Forces —

Mass no. A
Nuclear charge Z

isotopes : same Z, different A
isobars : same A, different Z

Chemical behaviour depends only on Z.
(Very slight dependence on A — only detectable by very refined chemical me[thods]
H, D } separated chemically)
N^{14} N^{15} }

Stable nuclei do not emit ε± — about 300 of them
A : from 1 — 238 (possibly $^{239}_{93}$).
Z : 1 — 92

Z = 43, 61, 85, 87 have not been observed in stable state
(Masurium) (Illinium) — no serious claims to discovery
Florentium
Claims to discovery false.
No reason why every element should have a stable representation —

A : 1 - 212 all exist in at least one stable representative
except 5 & 8. $Be^8 \to 2 He^4$ energy eff. sohw
unstable for 5 —
$He^5 \to He^4 + n + 1$ Mev unstable
Li^5 $He^4 + p + >1$ Mev more "
all others have at least one stable representation.

H	1 2
H	3 4
Li	6 7
Be	(8) 9
B	10 11
C	12 13
N	14 15
O	16 17 18
F	19
P	31
S	32 33 34
Cl	35 37
A	36 38 40
K	39 41 [40]
Ca	40 42 43 44

here even Z has 3 isotopes, odd Z only 1.

A^{40} & Ca^{40} first isobars.

Z protons

$N = A - Z$ neutrons

isotopic number: $A - 2Z = I$

isobars: usually even Z's, even A.
about 30 (or more) such pairs.

4 years ago:

(stable):
- 154 isotopes of even A, even Z
- 52 .. .. odd A, odd Z
- 55 .. .. odd A, even Z
- 4 .. .. even A, odd Z.

last 4: $_1H^2$ $_3Li^6$ $_5B^{10}$ $_7N^{14}$

(first four that could occur)

∴ odd A: either even / odd } Z equally likely

even A: always even Z except above 4.

even Z & N — most stable

even Z, odd N } not so stable
odd Z, even N }

odd Z odd N, almost all unstable.

isobars: even A, even Z's differing by 2 units (usually).
about 50 such pairs.

odd A Z's differing by 1 unit very rare.

Radioactive $_{37}Rb^{87}$ $_{38}Sr^{87}$
$_{48}Cd^{113}$ $_{49}In^{113}$
$_{49}In^{115}$ $_{50}Sn^{115}$
$_{51}Sb^{123}$ $_{52}Te^{123}$

(spurious ones claimed by Aston disproved by Bainbridge —)
(formation of hydrides balls the works)

Canadian mineral containing alkalis, but no alkali earths except Sr — It was suspected that $Rb^{87} \rightarrow Sr^{87} + e^-$ —

Hahn separated Sr from mineral & analyzed it in mass spectrograph — had only Sr^{87} whereas ordinary Sr has 4 isotopes — ∴ Sr from mineral did come from Rb decay).

Rb^{87} has very long lifetime ∴ must be comparable to age of earth (else there wouldn't be any left). estimated 10^{10} years half-life.

Large difference in nuclear spin between radioactive element & product means long life —

~~Assume~~ these isobars are radioactive with very long life —

∴ No ~~stable~~ isobars with $\Delta Z = 1$ unit;

∴ In general if we have a nucleus A, predict ~~that~~ probably no isobar with ΔZ 1 more likely one of $\Delta Z = 2$

2/28/40

276 natural nuclei $A \leq 209$

109 odd A		167 even A.	
105 possible values of A		104 possible values of A	
1 not occupied	5	1 not occupied	8
99 singly "		45 singly occupied	
5 doubly "	87, 113, 115, 123, 187	52 doubly "	
		6 triply "	(40, 96, 124, 130, 136, 176)
54 even Z		161 even Z	
55 odd Z			

40 : ^{18}A $^{19}K \to Ca^{20}$ 96 : ^{40}Zr ^{42}Mo ^{44}Ru

176 : ^{70}Yb $^{71}Lu \to Hf^{72}$.

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2 elements of odd Z ~~that~~ don't exist ($Z \leq 83$)
43, 61.

2 elements of even Z with no ~~odd~~ isotope of odd A
^{18}A, ^{58}Ce

43: 95 ^{42}Mo
97 ^{42}Mo hence no; ^{43} would decay
99 ^{44}Ru

similarly: 61: 145 ^{60}Nd ^{61} would decay.
147 ^{62}Sm

Relative abundance : Sn

112	1.1	119	9.8
114	0.8	120	28.5
115	0.4	122	5.5
116	15.5	124	6.8
117	9.1		
118	22.5		

even more abundant –

Energy. Can determine absolute energy content of nucleus by weighing; which can't be done with atoms –

(Weight of ~~atom~~ nucleus – wt. of constituents) c^2 = binding energy.

Conservation of energy has been so often checked in nuclei that we can use it to predict new facts —

Mass spectrograph used to determine mass; gives e/m of particle; ∴ necessary to know degree of ionization, then mass is known.

Either: compare ~~different~~ ~~same~~ approx same ratio e/m A's with same ionization;

~~or: " same A's : different "~~

e.g.: compare $\frac{1}{2}$ D^+ with H_2^+ $\frac{1}{2}$

or $\frac{1}{4}$ He^+ with D_2^+ $\frac{1}{4}$

$\frac{2}{12}$ C^{++} with D_3^+ $\frac{1}{6}$

$\frac{1}{16}$ $^{12}CH_4^+$ O^+ $\frac{1}{16}$

Bainbridge & Jordan 1936-37

Mattauch

Aston

Bonkas P.R. 55 696

atomic masses.

nuclear mass + mass of electrons in neutral atom.

n :	1.008 93
H :	1.008 12
D :	2.014 72
H^3 :	3.017 04
He^3 :	3.017 01
He^4 :	4.003 88
Li^6 :	6.016 90
He^6 :	6.020 90
⋮	
C^{12} :	12.003 98
O^{16} :	16.000 00
⋮	

Atomic masses because it's only way one can tell whether isobars are stable or unstable.

Stability of isobars:

suppose we have: Z^A and $(Z+1)^A$

transform into other by β-disintegration;

$$Z^A \rightarrow (Z+1)^A + \beta^- + \nu \text{ (neutrino)}$$

energetically possible if $Z^A > (Z+1)^A + m$ (+ rest mass of neutrino ~ 0)

then Z^A is unstable, $(Z+1)^A$ stable.

$$\left(Z^A + Zm > (Z+1)^A + m + Zm \right.$$
$$\left. \therefore M_{Z^A} > M_{(Z+1)^A} \text{ when } M\text{'s are atomic masses} \right)$$

or: $(Z+1)^A \rightarrow Z^A + \beta^+ + \nu$ (perhaps).

more likely: $(Z+1)^A + e^-_K \rightarrow Z^A + \nu$ (K-electron capture).

if $(Z+1)^A > Z^A + m$

if $(Z+1)^A - m > Z^A$ ∴ (this more likely)

$$\left(M_{Z+1} > M_Z \right)$$

∴ of two isobars, one with smaller M is stable.

one " larger M will either emit β^-, or absorb electron — is unstable.

H^3 should be unstable: $H^3 \rightarrow He^3 + \beta^- + \nu$

He^6 " " " : $He^6 \rightarrow Li^6 + \beta^- + \nu$

Some masses can be determined only by disintegration data —
e.g. neutron, H^3, He^3 neutron wouldn't work ; H^3 unstable,
He^3 too rare –

3/1/40
!!

To use disint. data :

1) must know kin. en. of incident particles
2) Measure accurately range of product of reaction
3) Range-energy relation – (from collision theory)
from measurements on α-particles ; for α > 12 Mev
p > 3 ...
must derive ~~theoretically~~

Liv. + Bethe
Rev. Mod. Phys.
9

4) Geometry

must know θ.

$$\underline{p}_{recoil} = \underline{p}_{inc.} - \underline{p}_{product}$$

and $$p_{rec}^2 = p_{in}^2 + p_{prod}^2 - 2\, p_{in} p_{prod} \cos\Theta$$

$$E_{rec} = \frac{p_{rec.}^2}{2M_{rec}} = \frac{M_{in} E_{in} + M_{pr} E_{prod} - 2\sqrt{M_i M_p E_i E_p}\cos\theta}{M_{rec.}}$$

$$Q = E_{pr} + E_{rec} - E_{inc.}$$

Thin targets : ones in which energy loss is within desired accuracy of energy measurement – almost impossible with solid targets ; possible with gaseous targets —

Solid targets must be considered thick; then a very thick one is desirable (correction difficult for medium thick target), so that product particle is completely absorbed; from knowledge of cross-section – energy relation of stuff, can finally calculate Q –

Oliphant + Rutherford '3[illegible] P.R.S.

$$ {}^1H^2 + {}^1H^2 = \begin{cases} {}^1H^3 + {}^1H^1 + 3.98 \text{ Mev.} & \pm .01_5 \\ {}^2He^3 + {}^0n^1 + 3.28 \text{ "} & \pm .07 \end{cases} $$

n ..% – 1.3 Mev – Bonner – but Bethe doesn't believe because no γ-rays emitted

To measure n, must use recoil protons –

$$E_n = \frac{E_p}{\cos^2\theta}$$

Allison – electric deflection

adjust potential so that product of reaction can go through –

good only up to 2 Mev. accurate to a few kilovolts

$$Be^9 + H^1 \rightarrow Be^8 + H^2$$

Binding energy of H^2, from:

$$H^2 + \gamma \rightarrow H^1 + n^1$$

ThC'' 2.62 Mev. | $.22_5$ Mev ±.03 ← same, since γ-ray momentum is negligible –

∴ total: .45 Mev ±.06 –

P.R.S.

Chadwick + Goldhaber '3[illegible]

", Feather + Bretscher '3[illegible]

cross section:

Von Halban + Kowarski Nature '38

$\therefore$ Binding energy of H^2 : $2.62 - 0.45 = 2.17 \pm .06$ Mev.

From this, find mass of n^1

From mass spectr. : $2H - H^2 = 1.53$ m.m.u.

$= 1.42$ M.e.v.

$H + n - H^2 = 2.17$

$\therefore \quad H - n = -.75$

$n - H = .75$ M.e.v.

Calculate difference in atomic weight of He^3 + H^3.

3/15/40 Wave f'n of deuteron over most of space: $e^{-(r-a)\alpha}$

Scattering of neutrons by protons.

Deuteron eq'n: $$\frac{d^2 u_0}{dr^2} + \left(\frac{mE}{\hbar^2} - \frac{mV}{\hbar^2}\right) u_0 = 0 \qquad l = 0$$

Scattering (Differential) Cross Section:

$$d\sigma = \left| \frac{1}{2ik} \sum_l \left(e^{2i\delta_l} - 1\right)(2l+1) P_l \cos\theta \right|^2 2\pi \sin\theta$$

$k = \frac{\sqrt{mE}}{\hbar}$ $\quad \delta_l$ very small for ordinary neutron energies, $l \neq 0$

distance at which particles pass each other:

$$\sim \lambda l \qquad \lambda = 1/k$$

For $l = 1$, δ_1 small, if $\lambda > a$ $\quad$ a - range of force, $a \sim 2 \times 10^{-13}$

$$\lambda = \frac{\hbar}{\sqrt{mE}} = \sqrt{\frac{41.5 \text{ Mev.}}{E}} \cdot 10^{-13} \text{ cm.}$$

$\therefore$ $\lambda > a$ if $E < 10$ Mev

For most scattering expts, $E \sim 2, 4$ Mev;

$\therefore$ only ~~δ_0~~ δ_0 appreciable

Calculate δ_1 for rectangular potential with range $a = 2 \times 10^{-13}$ cm

get numerical values for $E_0 = 2$ & for 15 Mev.

E is energy in c. of m. coordinates; $= \frac{1}{2} E_0$ (en. in lab. system)

${}_3Li^7 + {}_1H^2 \to {}_4Be^8 + {}_0n$ $\quad$ 15 Mev

$H^2 + H^2 \to He^3 + n$ $\quad$ 2 Mev

if: only $l = 0$, Spherical symmetry (in c.g.m. system

in lab. system: scattered neutrons as $\sin\theta\cos\theta\, d\theta$.

Experimental difficulties: a) scattered neutrons

b) discrimination against short range protons (at large angles)

$E_{proton} = E_{neutron} \cos^2\theta$. ⟵

Cambridge Dee + Gilbert '38 P.R.S.

Illinois Kruger, Shoupp '38 P.R.

Established that spherically sym. scatt. for 2.5 Mev neutrons

for only δ_0;

$$d\sigma = \left|\frac{1}{2ik}\left(e^{2i\delta_0}-1\right)\right|^2 2\pi \sin\theta\, d\theta$$

$$= \left|\frac{1}{2ik}\left[1+2i\delta_0-1\right]\right|^2 2\pi \sin\theta\, d\theta$$

$$= 2\pi\left(\frac{\delta_0}{k}\right)^2 \sin\theta\, d\theta$$

$$\sigma = 4\pi\left(\frac{\delta_0}{k}\right)^2$$

for larger δ_0: $\left|e^{2i\delta_0}-1\right|^2 = 2(1-\cos 2\delta_0) = 4\sin^2\delta_0$

and $\sigma = 4\pi\left(\frac{\sin\delta_0}{k}\right)^2$.

Rectangular ~~potential~~: $\gamma^2 = \frac{mV_0}{\hbar^2}$

$r < a$: $\frac{d^2u}{dr^2} + (k^2+\gamma^2)u = 0$

$$u = c_1 \sin\left(\sqrt{k^2+\gamma^2}\right) r$$

$r > a$: $u'' + k^2 u = 0$

$$u = c_2 \sin kr + c_3 \cos kr$$

$$= c_4 \sin(kr + \delta_0)$$

match at $r = a$:
(to find δ_0).
$$\frac{1}{u}\frac{du}{dr} = \sqrt{k^2+\gamma^2}\,\cot (k^2+\gamma^2)^{1/2} a = k \cot(ka+\delta_0)$$

if $k \ll \gamma$ (which means $E \ll V_0$) : $\gamma \cot \gamma a = k \cot(ka+\delta_0)$

Ground state : $\sqrt{\gamma^2-\alpha^2}\cot\sqrt{\gamma^2-\alpha^2}\,a = -\alpha$

(since $\alpha^2 \ll \gamma^2$) or $\gamma \cot \gamma a = -\alpha$

$$\therefore\ -\alpha = k \cot(ka+\delta_0)$$

also ka is small; $ka \ll \gamma a \sim \frac{\pi}{2}$.

$\therefore$ Neglect it.

$$\therefore\ -\alpha = k \cot \delta_0$$

$$\cot\delta_0 = -\frac{\alpha}{k}.$$

$$\therefore\ \sigma = 4\pi \frac{\sin^2\delta_0}{k^2}$$

$$= 4\pi \frac{1}{k^2(1+\cot^2\delta_0)}$$

$$= \frac{4\pi}{k^2\left(1+\frac{\alpha^2}{k^2}\right)} = \frac{4\pi}{k^2+\alpha^2}$$

$$= 4\pi \frac{\hbar^2}{m}\frac{1}{\epsilon + E} = \frac{4\pi\hbar^2}{m\left(\epsilon + \frac{E_0}{2}\right)}.$$

ϵ = B.E. of deuteron $= \frac{\hbar^2}{m}\alpha^2$

$$\sigma = \frac{4\pi \cdot 41.5\text{ Mev}}{\epsilon + \frac{1}{2}E_0} \cdot 10^{-26} = \frac{5 \times 10^{-24}}{2.17} = 2.3 \cdot 10^{-24}\ \text{cm}^2$$

if $E_0 \ll \epsilon$: observed: $15\text{-}18 \cdot 10^{-24}\ \text{cm}^2$

∴ Theory doesn't give correct results for slow neutrons, tho it does for fast.

3/20/40 | Cross section for neutrons scattered by protons:

$$\sigma = \frac{4\pi}{\alpha^2 + k^2} = 4\pi \cdot \frac{\hbar^2}{m} \cdot \frac{1}{\varepsilon + \frac{1}{2}E_0}$$

$$E_0 = 0, \quad \sigma = 2.3 \times 10^{-24}\ \text{cm}^2$$

Measurements for small E_0 give $\sigma = 15\text{-}18 \times 10^{-24}\ \text{cm}^2$

Discrepancy can't be removed by finite range of nuclear forces;

Wigner 1935 gave solution;

p, n each have spin ½;

p + N ∴ has spin 0 or 1.

↓ singlet state ↓ triplet state. (because of splitting in magnetic field)

Ground state: spin = 1, ~~triplet~~.

∥ spin

orbital momentum = 0, ∴ S state

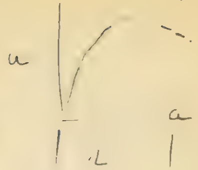
∴ total angmom = 1 ∴ Ground state: $^3S_1$

possible α has different value for singlet + triplet states;

$-\alpha = \frac{1}{u}\frac{du}{dr}$ at $r=a$, for triplet state.

Then $\left(\frac{1}{u}\frac{du}{dr}\right)_{r=a} = \beta$, singlet state;

if $\beta > 0$, u increases at a;
$\beta < a$, u decreases at a;
$\beta > 0$, no stable singlet state; ("virtual state")



but if $\beta < 0$, outside a, u: decreasing exponential

$\therefore$ stable singlet state, $B.E. = \frac{\hbar^2}{m}\beta^2$;
$= \epsilon'$

Then $\sigma_{singlet} = \frac{4\pi}{\beta^2 + k^2}$

$= 4\pi \frac{\hbar^2}{m} \frac{1}{\epsilon' + \frac{E_0}{2}}$

If many neutrons are falling on a proton, some will have ∥ spin, some anti-∥; in fact, 3/4 parallel,

& 1/4 anti-p., since weight of ^3S state is 3.

$\therefore \sigma_{obs} = \frac{\sigma_{single}}{4} + \frac{3\sigma_{trip}}{4}$

$= \pi \frac{\hbar^2}{m}\left(\frac{1}{\epsilon' + \frac{E_0}{2}} + \frac{3}{\epsilon + \frac{E_0}{2}} \right)$

for $\sigma_{obs} = 16.8 \times 10^{-24}$, $\epsilon' = \frac{1}{25}\epsilon$,

and $\beta' = \frac{1}{5}$ ~~α~~.

(still don't know whether β is + or −).

Interference of neutrons scattered from two protons

two protons, as in H_2^{m} have spins either ∥ or anti-∥.

incoming neutron, also either 11, or anti-11;

if 11, triplet scattering from each; } for ortho H_2, $(\uparrow\uparrow)$
if anti-11 singlet " " "; }

∴ constructive interference. — large cross-section.

for para H_2 $(\uparrow\downarrow)$, triplet scattering from one & singlet from other. call amplitudes a & b.

interference; scattered amplitude, a + b; if they have same sign, large intensity, different sign, small intensity. ∴ constructive interference if $\beta < 0$ (real singlet state)
destructive " if $\beta > 0$ (virtual state)

More exact treatment.

spin functions σ ($m_s = +\frac{1}{2}$), τ ($m_s = -\frac{1}{2}$)

$$\text{Triplet}: \begin{cases} \sigma(n)\,\sigma(p) = \chi_1 \\ \frac{1}{\sqrt{2}}\left[\sigma(n)\,\tau(p) + \tau(n)\,\sigma(p)\right] = \chi_0 \quad 2 \\ \tau(n)\,\tau(p) = \chi_{-1} \quad 3 \end{cases}$$

$$\text{Singlet}: \frac{1}{\sqrt{2}}\left[\sigma(n)\,\tau(p) - \tau(n)\,\sigma(p)\right] = \chi_s \quad 4$$

Define: scattering operator S
S applied to spin function of original system gives spin function of scattered beam.

$$S \sum_1^4 c_n \chi_n = a \sum_1^4 c_n \chi_n + b\, c_s \chi_s .$$

$\cot \delta_0 = \frac{\beta}{k}$. (scattering $\propto e^{2i\delta_0} - 1$).

if $\beta > 0$, $\delta_0 \to 0$ as $k \to 0$ and $e^{2i\delta_0} - 1 \to 2i\delta_0 > 0$

if $\beta < 0$, $\delta_0 \to \pi$ as $k \to 0$ and $e^{2i\delta_0} - 1 \to 2i(\delta_0 - \pi) < 0$

$\therefore\ b > 0$ for $\beta > 0$ (virtual state)

$b < 0$ for $\beta < 0$ (real state)

$a < 0$.

Para H_2: $\frac{1}{\sqrt 2}\left(\sigma(1)\tau(2) - \sigma(2)\tau(1)\right)$

$$\chi = \frac{1}{\sqrt2}\left[\sigma 1 \tau 2 - \sigma 2 \tau 1\right] \sigma(n) .$$

$$S\chi = \frac{1}{\sqrt2}\left[\tau_2 S_1 \sigma_{(1)}\sigma(n) - \sigma_2 S_1 \tau_1 \sigma_n\right]$$

~~triplet~~ + similar S_2

$$= \frac{1}{\sqrt2}\Big[\tau_2\, a\, \sigma(1)\sigma(n) - \sigma(2)\Big\{\tfrac12(a+b)\sigma(n)\tau(1) + \tfrac12(a-b)\tau(n)\sigma(1)$$

$$+ \sigma(1)\Big[\tfrac12(a+b)\tau(2)\sigma(n) + \tfrac12(a-b)\sigma(2)\tau(n)$$

$$- \tau(1)\sigma(2)\sigma(n)$$

$S\ \tau(1)\sigma(n)$

$= \frac{1}{\sqrt2}\left(a\chi_0 + b\chi_3\right)$

$= \frac12(a+b)\sigma(n)\tau(1) + \frac12(a-b)\tau(n)\sigma(1)$

$$= (3a+b)\chi + (a-b)\chi_{\text{trip}}(1,2)\ \sigma_{\tau}(n) .$$

$\uparrow$ elastic scattering

$\uparrow$ ~~transition~~ from ortho $\to$ para H_2

If energy of neutron is too small to make ortho → para H_2, only elastic scattering will take place;

for a ~~virtual state~~ $b > 0$, $|b| = 5 \cdot 10^{-13}$,

and $S\chi = -2a\chi$

for real state, $S\chi = 8a\chi$.

Scattering of neutrons by para H_2 will be 4 times as large (& same order as ortho H_2 scattering) ~~if~~ for ~~virtual~~ real state as for virtual;

Experiment: small scattering for para H_2;

∴ singlet state is virtual ($b < 0$, $\beta < 0$)

3/22/40 | Scattering depends on state of nucleus.

Scattering by a number of particles bound by forces;

Classically, scattering independent of forces, if time of collision is small compared with periods of system (as it usually is). Then energy is distributed among particles of system.

Q. M'elly ; time is longer - so more complicated -

Particular form of neutron potential doesn't affect results, since range is so much smaller than wave length of molecular wave fn's ∴ replace it by longer & lower potential (still small relative to molecular wave lgth) & apply Born approximation.

$$\sigma = \frac{\mu^2}{\pi \hbar^4} \left| \int V \, d\tau \right|^2$$

1) particle ~~tightly~~ bound : $\mu = m \cdot 1$

2) particle free : $\mu = \frac{m M}{m + M} = 1/2$.

∴ cross section of bound particle 4 times as much.

Differential cross section for ~~transition~~ from state m to n;

$$\sigma_{mn} = \left| \frac{m}{2\pi \hbar^2} \int \psi_m^*(\underline{r}_p) \, e^{-i k' \underline{r}_n} \, V(\underline{r}_p - \underline{r}_n) \, \psi_n(\underline{r}_p) \, e^{i k \underline{r}_n} \, d\tau_p \, d\tau_n \right|$$

ψ_m^* → proton final; $e^{-ik'r_n}$ → neutron; V → potential; ψ_n → pr. initial state; e^{ikr_n} → neutr.

cross section for free proton :

$$Q = \frac{m^2}{4\pi \hbar^4} c^2$$

$$\therefore c = \frac{\hbar^2}{m} \sqrt{4\pi Q}$$

Let $V = c \, \delta(\underline{r}_p - \underline{r}_n)$

$$\therefore V = \frac{\hbar^2}{m} \sqrt{4\pi Q} \, \delta(\underline{r}_p - \underline{r}_n).$$

$$\therefore \sigma_{mn} = \left| \sqrt{\frac{Q}{\pi}} \int \psi_m^*(\underline{r}_p) \, e^{i k'' \underline{r}_p} \, \psi_n(r_p) \, d\tau_p \right|$$

$k - k' = k''$

$$= \frac{Q}{\pi} \left| \left(e^{i k \underline{r}_p} \right)_{mn} \right|^2$$

(q = cross section / unit solid angle) $= 4q \left| \left(e^{i k \underline{r}_p} \right)_{mn} \right|^2$

If several protons, sum over all of them in molecule -

Suppose proton in isotropic oscillator;

$$\psi = \psi_x \psi_y \psi_z$$

$$\psi_0 = e^{-\frac{1}{2}\left(\frac{x}{a}\right)^2} a^{-1/2} \pi^{-1/4}$$

$$\therefore \quad \sigma_{00} = 4q \left| \left(e^{i h_0 r} \right)_{00} \right|^2$$

$$= 4q \left| \frac{a^{-1}}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{i h'' x - \left(\frac{x}{a}\right)^2} dx \right|^2$$

$$= \frac{4q a^{-2}}{\pi} \left| a \int_{-\infty}^{\infty} e^{i b y - y^2} dy \right|^2$$

$h'' = 2k \sin \theta/2$

$$= \frac{4q}{\pi} \cdot \left| \sqrt{\pi} \cdot e^{-\frac{b^2}{4}} \right|^2$$

$b = a h''$

$= 2ak \sin \theta/2$

$$= 4q\, e^{-\frac{b^2}{2}}$$

$$= 4q\, e^{-2(ak)^2 \sin^2 \theta/2}$$

$$\sigma_{obs} = \sigma \cdot \frac{v_{rel} \text{ to part}}{v_{observer} \text{ rel. to}}$$

3/27/40 | Photo~~electric~~ effect of D.

← annoying hard!

cross section for any photo-el. effect:

$$\sigma = \frac{4\pi^2 \omega}{c} \left| M_{0E}^{el} \right|^2 .$$

(imagine light wave disturbing system; calculate ~~transition~~ probability.) Wave f'n of excited state normalized per unit energy.

For deuteron: $M_{0E}^{el} = e \int \psi_0 \psi_E \, d\tau \cdot \frac{1}{2} z$

↑ coordinate of proton w. respect to neutron ($\frac{1}{2} z$ – w. resp. to c. of m.)

$r > a$: $\psi_0 = \frac{1}{\sqrt{\pi \alpha}} \frac{e^{-\alpha r}}{r}$

$$\psi_E = e^{i \underline{k} \cdot \underline{r}} \cdot \frac{1}{\sqrt{\Omega}} \sqrt{\frac{\Omega \cdot 4\pi p^2 dp}{h^3 dE}}$$

($l = 1$ since only ~~transition~~ to P state will be allowed, on acct of factor z)

might as well write a plane wave

Ω: volume of box;

$\sqrt{\ }$: no. of qu. states per unit energy in ~~that~~ box.

but $\frac{dE}{dp} = v$ $\quad p = \frac{1}{2} M v$ $\quad$ ($\frac{1}{2}$ because rel. coörds)

$$\therefore \psi_E = e^{i \underline{k} \cdot \underline{r}} \sqrt{\frac{4\pi \cdot p^2 M}{h^3 \cdot 2p}} = e^{i \underline{k} \cdot \underline{r}} \sqrt{\frac{M p}{4\pi^2 \hbar^3}}$$

$$\therefore M_{0E}^{el} = \frac{e \sqrt{Mp}}{4\pi \hbar^{3/2} \sqrt{\pi \alpha}} \int d\tau \, \frac{e^{-\alpha r}}{r} z \, e^{i \underline{k} \cdot \underline{r}}$$

$$= \frac{1}{i} \frac{\partial}{\partial k_3} \int d\tau \, \frac{e^{-\alpha r}}{r} e^{i \underline{k} \cdot \underline{r}}$$

$$= 2\pi \int r^2 \, dr \; \frac{e^{-\alpha r}}{r} \left(\frac{e^{ikr}}{ikr} + \text{conj} \right)$$

$$= \frac{2\pi}{ik} \frac{1}{\alpha - ik} + \text{conj}.$$

$$= \frac{2\pi}{ik} \left[\frac{1}{\alpha - ik} - \frac{1}{\alpha + ik} \right]$$

$$= \frac{4\pi i k}{ik(\alpha^2 + k^2)} = \frac{4\pi}{\alpha^2 + k^2}.$$

$$\frac{1}{i} \frac{\partial}{\partial k_3} \left(\int \right) = -\frac{4\pi}{i} \frac{2k_3}{(\alpha^2 + k^2)^2}$$

$$\therefore M_{0E}^{el} = -\frac{e\sqrt{Mp}}{i\hbar^{3/2}\sqrt{\pi\alpha}} \frac{2k_3}{(\alpha^2 + k^2)^2}$$

$$|\;|^2 = \frac{4e^2 Mp}{\pi\alpha \hbar^3} \frac{1}{3} \frac{k^2}{(\alpha^2 + k^2)^4}$$ average over all directions.

$$\therefore \quad \sigma = \frac{16 e^2 \pi \omega M p}{3c\alpha \hbar^3} \frac{k^2}{(\alpha^2 + k^2)^4}.$$

$$= \frac{2\pi}{3} \cdot \frac{e^2}{\hbar c} \cdot \frac{\hbar^2}{M} \cdot \frac{\sqrt{\epsilon}\, E^{3/2}}{(E + \epsilon)^3}$$

ϵ : B.E. of deuteron

E . en. of

$$= 1.16 \cdot 10^{-26} \cdot \frac{(\gamma - 1)^{3/2}}{\gamma^3}$$

$$\gamma = \frac{\hbar\omega}{\epsilon} = \frac{E + \epsilon}{\epsilon}.$$

Scattering cross-section was:

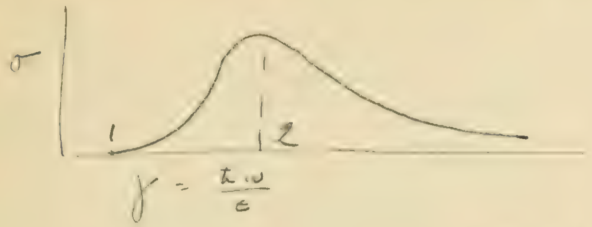
$$\sigma = \frac{4\pi \hbar^2}{M(\epsilon + E)}$$ (triplet st.)

γ-ray less than ϵ energy, no effect.

if greater, excess energy goes to kin. en. of p. + neutr.

cross-section has a max when ?

at very high freq, σ decreases again;



$$\sigma_{max} = 1.4_5 \cdot 10^{-27}\ cm^2 .$$

radiation effects are small compared to (only) particle effects;

$\frac{e^2}{\hbar c}$ always comes in; even less than that factor, though;

Measurements made at $\hbar\omega = 2.62$ Mev (Th C'' γ's).

$$\epsilon = 2.17 ,$$

$$\therefore \gamma = 1.2 ,$$

To compare with exp., must consider wave functions in $r < a$ as well (nuclear force range). Multiplies σ by factor of ~~about~~ 2. (Breit & Condon, P.R. 1936).

They get $\sigma = 7 \cdot 10^{-28}$ for the C'' γ's,

observed same, by Halban & Kowarski, Nature ?

Prob. of ~~emission~~ in ~~a given direction~~ ~~direction of incidence~~ ~~is proportional to~~ a given direction is prop to $\sin^2\theta$,

where Θ is angle betw. polarization of incident radiation, & emitted proton;

For unpolarized γ-rays;

Proton scattering $\sim l_y^2 + l_z^2 \sim \sin^2 \Theta$.

(incident radiation is x-direction) (Θ = angle betw. x & proton)

(verified - more or less.

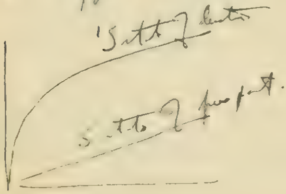
Photo-magnetic effect.

$$\sigma = \frac{4\pi^2 \omega}{c} \left| M_{0E}^{Magn} \right|^2$$

$$M = \sum_{spins} \int \psi_E^* \mu_z \psi_0 \, d\tau \qquad \mu \text{ - magn. mom't operator}$$

$$\mu = e\frac{\hbar}{Mc}. \quad M \sim e a \qquad a \gg \frac{\hbar}{Mc}; \quad \therefore \text{ magnetic effects smaller than elec.}$$

photo-mag effect can give a transition from ground S-state to excited S-state, which elec. effect can't.



3/29/40 $\mu_z = \mu_p \sigma_{zp} + \mu_n \sigma_{zn}$ σ – Pauli operators

$\mu_p = 2.718$, $\mu_n = -1.953$ } nuclear magneton

one " " $= \frac{e\hbar}{2Mc}$.

$$\psi_0 = \frac{e^{-\alpha r}}{\sqrt{\pi\alpha}\, r} \cdot \frac{1}{\sqrt{2}}\left[\sigma(1)\tau(2) + \tau(1)\sigma(2)\right]$$ } triplet

($\sigma_1 \sigma_2$; $\tau_1 \tau_2$)

$$\psi_E = \psi_E(r)\ \frac{1}{\sqrt{2}}\left[\sigma(1)\tau(2) - \tau(1)\sigma(2)\right]$$ – singlet.

1 – p
2 – n

$$M = \int d\tau\ \psi_0(r)\psi_E(r) \cdot \sum_\sigma \frac{1}{2}\left[\sigma(1)\tau(2) - \tau(1)\sigma(2)\right]\left\{\left[\mu_p - \mu_n\right]\sigma(1)\tau(2) + (\mu_n - \ldots\right.$$

$\tau(1)\sigma \ldots$

$$= \sum_\sigma \frac{1}{2}\left[\chi_{\sigma_{sing}}\ [\mu_p - \mu_n]\ \chi_{sing}\right.$$

$$= (\mu_p - \mu_n)$$

$(\mu_z \chi_{trip} = (\mu_p - \mu_n)\chi \ldots$

Normally, $\int \psi_0 \psi_E = 0$;

But here one is singlet & other triplet; (Both s-states)

Proof of orthogonality rests on equality of pot. en. in the two cases; here forces are entirely different; $\therefore \psi_0$ & ψ_E are not orthogonal.

$$\psi_{E\,l=0} = \frac{\sin(kr+\delta)}{kr}\sqrt{\frac{4\pi p^2 dp}{h^3 dE}} \quad r > a$$

$$\tan\delta = \cancel{\frac{\beta}{k}} = \frac{k}{\beta}$$

$$\int d\tau = \sqrt{\frac{M p}{2\pi^2 \hbar^3}} \cdot \frac{4\pi}{\sqrt{\pi\alpha}}\frac{1}{k}\int \frac{1}{r^2} e^{-\alpha r}\sin(kr+\delta)\, r^2 dr$$

$$\left(= Im \int e^{-\alpha r + ikr + i\delta}\, dr \right.$$

$$= Im\ \frac{e^{i\delta}}{\alpha - ik} = \frac{k\cos\delta + \alpha\sin\delta}{\alpha^2 + k^2}$$

$\cos\delta = \frac{\beta}{\sqrt{k^2+\beta^2}}$ $\sin\delta = \frac{k}{\sqrt{k^2+\beta^2}}$

$\therefore \int : \frac{k(\beta+\alpha)}{(\alpha^2+k^2)\sqrt{\beta^2+k^2}}$. $\begin{matrix} \tfrac{6}{5}\alpha \\ \tfrac{4}{5}\alpha \end{matrix}\left(\begin{matrix} \alpha+|\beta| & \text{if singlet state virtual} \\ \alpha-|\beta| & \text{if real.} \end{matrix}\right.$

from ~~exp~~, $|\beta| \sim \frac{1}{5}\alpha$; ∴ ↑

∴ 3/2 as much for virtual state;

and cross-section, 9/4 or over times as great.

$$\sigma_{virtual} \sim \frac{9}{4}\sigma_{real}$$

Experiment again indicates virtual state.

if trip & singl state have same B.E., $\alpha = \beta$, and $\alpha - |\beta| > 0$;

(for then the wave f'ns are orthogonal).

$g = \mu$

$$\therefore \quad \sigma = \frac{4\pi^2\omega}{c} \cdot \frac{8}{\pi} \cdot \frac{Mp}{\alpha k^3} \frac{(\beta+\alpha)^2}{(\alpha^2+k^2)^2(\beta^2+k^2)} e^2\left(\frac{\hbar}{2Mc}\right)^2(g_p - g_n)$$

4.7

$\alpha^2 + k^2 \sim \omega$

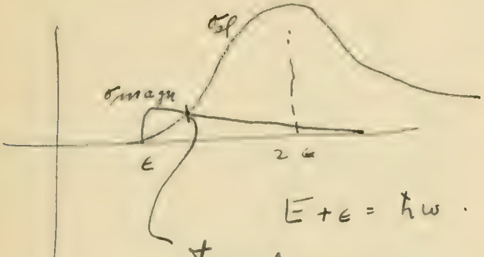
$$\sigma_{magn.} \sim \frac{E^{1/2}}{(E+\epsilon)(E+\epsilon')}$$

we had: $\sigma_{el} \sim \frac{E^{3/2}}{(E+\epsilon)^3}$

Must have g_p & g_n with opposite signs

$\sigma_{magn} << \sigma_{el}$, since c^3 in denom. for σ_{magn}, only c in denom for σ_{el}.

For small ~~large~~ energy, σ_{mag} becomes larger, compared with σ_{el}. For large E, same energy dependence.



$$E + \epsilon = \hbar\omega .$$

at 150 kv, $\sigma_{magn} = \sigma_{el}$.

Von Halban: $E \sim 500$ kv

angular dist: $\alpha \sin^2\theta + \beta$. $\beta \sim .2\alpha$
el magn

~~exp:~~ $\dfrac{I_{\theta=0}}{I_{\theta=90}} = \begin{cases} .02 \text{ exp.} \\ .2 \text{ ~~theoretical~~} \end{cases}$

one of the most disconcerting things in nuclear physics

Reverse effect: capture of neutron by proton.

Suppose: 2 effects; one, and its reverse.

$$A \rightleftarrows B .$$
(neutron) (γ-ray).

Cross-section for any process: $\frac{1}{v}$ (Emit density) · Number of final states per unit energy.

v incident.

$\sim \left(\dfrac{p^2 dp}{dE}\right)_{\text{final particle}}$

$\sim \dfrac{p^2}{v}$ final

$\therefore \sigma_{AB} \sim \frac{p^2{}_B}{v_A v_B} \sigma_{\text{intrinsic}}$

$\therefore \sigma_{BA} \sim \frac{p^2{}_A}{v_A v_B} \sigma_{\text{intrinsic}}$

$\therefore \frac{\sigma_{AB}}{\sigma_{BA}} \sim \frac{p^2{}_B}{p^2{}_A} = \frac{\lambda^2{}_A}{\lambda^2{}_B}$ ratio of wave-lengths of incident particles.

$\therefore \frac{\sigma \text{ capture}}{\sigma \text{ photoeff}} = \frac{p^2{}_{\gamma\text{-ray}}}{p^2{}_{\text{neutron}}} = \frac{(\hbar\,\omega/c)^2}{ME} = \frac{(E+\epsilon)^2}{Mc^2 E}$.

(c.g. of m. coordinates)

$\therefore \sigma_{\text{magn capture}} \sim \frac{E+\epsilon}{E+\epsilon'} \cdot \frac{1}{\sqrt{E}} \sim \frac{1}{v}$

Frisch, v. Halban, Koch : $3 \cdot 10^{-25}$ cm^2 for $E = kT$ room temp.

agrees well with theory.

4/19/40 ~~Photomagnetic effect at high energies~~

Proton – Proton scattering.

~~Can~~ [illegible] ~~due to~~ [illegible] nuclear forces.

White 1936 [illegible]
[illegible]
[illegible]

Scattering for Coulomb Field

$$\nabla^2\psi + (k^2 - \frac{\beta}{r})\psi = 0$$

$$k = \frac{Mv}{\hbar}$$ M reduced mass, v rel. vel.

$$\beta = \frac{2MZ_1Z_2e^2}{\hbar^2}$$

Bohr radius $a_0 = \frac{\hbar^2}{me^2}$;

$$\beta = 2\frac{M}{m}\frac{Z_1Z_2}{a_0}$$

for P Scattering $2M = M_p$, $Z_1 = Z_2 = 1$

Want solution $\beta = \frac{1}{a_p}$ $a_p = \frac{\hbar^2}{M_p e^2}$

which will be a plane wave at ∞ $\psi = e^{ikz}$

try a solution $\psi = e^{ikz}F$ where $F \to 1$ [illegible]

$$\nabla^2 F + 2ik\frac{\partial F}{\partial z} - \frac{\beta F}{r} = 0$$

Ψ = [illegible] $F = F(z \cdot y)$

([illegible])

$\Psi = \left(1 - \frac{[illegible]}{[illegible](z-y)}\right) e^{[illegible]}$

[illegible] $+ \frac{1}{2} e^{[illegible]} \frac{[illegible] e^2}{2 [illegible] \sin^2 \frac{\theta}{2}} e^{[illegible]}$

[illegible]

[illegible]

$\sigma(\theta) \sin\theta \, d\theta = 2\pi \left(\frac{[illegible]}{[illegible]} \right)^2 \frac{1}{\sin^4 \frac{\theta}{2}}$

[illegible];

1) [illegible]

Anti-sym. space fn. $\Psi(x y z) = -\Psi(-x, -y, -z)$

(rel. [illegible] coords)

$\therefore$ [illegible] $\Psi = \psi(1, 2) - \psi(2, -2)$ $F(\theta) \rightarrow F$ [illegible]

[illegible]

2) [illegible]

[illegible] Ψ [illegible]

$\sigma_{trip} \sim |F(\theta) - F(\pi-\theta)|^2$ weight 3

$\sigma_{singlet} \sim |F(\theta) + F(\pi-\theta)|^2$ weight 1.

$\therefore \sigma = \frac{3}{4}\sigma_{trip} + \frac{1}{4}\sigma_{singlet}$.

$= |F(\theta)|^2 + |F(\pi-\theta)|^2 - \mathrm{Re}\,(F(\theta) F^*(\pi-\theta))$

$= \frac{1}{\sin^4 \theta/2} + \frac{1}{\cos^4 \theta/2} - \frac{1}{\sin^2 \cos^2} \cos\left(\alpha \log\left(\frac{1+\cos\theta}{1-\cos\theta}\right)\right)$

if α is small cos( ) ~ 1;

and $\sigma = \sin^{-4}\theta/2 + \cos^{-4}\theta/2 - \sin^{-2}\theta/2 \cos^{-2}\theta/2$

angle Ψ in centre of mass system $= 2\,\Theta$ – lab system

$\therefore \sigma = \sin^{-4}\Theta + \cos^{-4}\Theta - \sin^{-2}\Theta\cos^{-2}\Theta$

[illegible] Θ [illegible] $\pi - \Theta$

[illegible]

for $\Theta = 45°$, $\sin\Theta = \cos = 1/\sqrt{2}$

and $\sigma = \frac{1}{2}$ [illegible]

Deviation from the [illegible]

from [illegible] At low energies, [illegible], at higher

[illegible]

4/12/40

Mott: $\left(\frac{Z_1 Z_2 e^2}{2mv^2}\right)^2 \left[\frac{1}{\sin^4\Theta} + \frac{1}{\cos^4\Theta} - \frac{1}{\sin^2\Theta\cos^2\Theta}\cos(\alpha \log\cot^2\Theta)\right]$

How modified if nuclear interaction exists also?

When we add nuclear potential to coulomb, we must use polar coordinates instead of ~~polar~~ parabolic; or at least we ~~can~~ must add a correction (in polar coord.) ~~to the~~ parabolic expansion; for small r:

$$\Psi(\text{coulomb} + \text{nuclear}) = \psi\,\text{coul} + \psi_s(\text{coul}+\text{nucl}) - \psi_s\,\text{coul}.$$

$$\frac{d^2\psi_s r}{dr^2} + \left(k^2 - \frac{\beta}{r} - \frac{2m}{\hbar^2}V(r)\right)(\psi_s r) = 0$$

For large r:

$$\psi_\ell(\text{coulomb}) = \frac{1}{kr}\sin(kr - \tfrac{1}{2}\ell\pi + \eta_\ell - \alpha\log 2k$$

$$\eta_\ell = \arg\Gamma(\ell+1+i\alpha)$$

$$= \arg\left[\Gamma_0 + \frac{1+i\alpha}{\sqrt{1+\alpha^2}} + \frac{2+i\alpha}{\sqrt{4+\alpha^2}} + \dots + \frac{\ell+i\alpha}{}\right]$$

$$= \eta_0 + \sum_\ell \arctan\frac{\alpha}{\ell}$$

$$\psi_\ell(\text{Coul}+\text{Nucl}) - \psi_\ell(\text{coul}) = \left(e^{2i\delta_\ell} - 1\right)\frac{1}{2ikr}e^{i(kr - \frac{\ell\pi}{2} + \eta_\ell - \alpha\log 2kr)}$$

δ_0 : phase shift $\qquad \sim \frac{\sin^2\delta_0}{k^2} \sim \frac{1}{E+\epsilon}$

at high energies, nuclear cross-section greater

at low " , coulomb " "

(since coulomb $\sim \frac{1}{E^2}$)

(also experimentally).

But at small angles, coulomb scatt. will predominate, no matter what energy, because of angular factor. At intermediate angles, interference term comes in; in proton-proton scattering, it helps detect nuclear-scattering of p-waves; Breit: $\delta_1 < .01$ radian, 2400 kv;

Interference also determines sign: $\delta_0 > 0$, ∴ attractive pot.

$\delta_0 > \pi/2$ means real state of He^2, which doesn't exist

∴ $0 < \delta_0 < \pi/2$ (for if $-\pi < \delta_0 < -\pi/2$, as $E \to \infty$, δ_0 would $\to -\pi/2$; but it doesn't).

δ_0 is only unknown in final scattering function; ∴ must be able to adjust it to experimental data, which has always been done. (Bethe thinks this proves Q-M valid for nuclear problems).

Having found δ_0, must determine nuclear potential.

δ_0 varies with energy (in Nu-P scatt, $\tan \delta_0 = \frac{k}{\alpha}$

For rectangular potential, nuclear range $a \approx 2.8 \cdot 10^{-13}$ cm.

For: $\dfrac{e^{-r/a}}{r/a}$ " " : $a \sim 1.3 \cdot 10^{-13}$.

(Yukawa)

$\lambda = \frac{\hbar}{\mu c}$ mass of meson, from Yukawa potential, $\mu \sim 330\,m$

" " " " cosmic rays, $\mu \sim 200\,m$

For a given range, depth can be calculated accurately; and agrees well with P-N singlet depth. (Bethe thinks this very fundamental).

Many calculations about He^3, H^3, He^4 etc — results negative

4/17/41

4/19/40

$D(u)$ = spacing of levels

$\rho(u) = \frac{1}{D(u)}$ = level density —

$\rho(E)\, e^{-E/T}$ can make this vanish except for $E \sim U$

$$U = \frac{\sum_k E_k e^{-E_k/T}}{\sum_k e^{-E_k/T}} = \frac{\int E\,\rho(E)\, e^{-E/T} dE}{\int \rho(E)\, e^{-E/T} dE}$$

$$e^{-F/T} = \sum e^{-E/T} = \int \rho(E)\, e^{-E/T} dE .$$

$$-\frac{d\, e^{-F/T}}{d(\frac{1}{T})} = F e^{-F/T} + \frac{1}{T} e^{-F/T} \frac{dF}{d(\frac{1}{T})} = \int E\,\rho(E)\, e^{-E/T} dE$$

$$= e^{-F/T}\left(F - T\frac{dF}{dT}\right)$$

$$\therefore \quad U = F - T\frac{dF}{dT} .$$

$\therefore$ F is like free energy in thermodynamics,

then $S = -\frac{dF}{dT}$, S → entropy.

$$U = F + TS .$$

$$e^{-F/T} = \int \rho(E)\, e^{-E/T} dE = \frac{\rho(u)\, e^{-u/T}}{\lambda(u)}$$

$$\therefore \quad \rho(u) = \lambda(u)\, e^{\frac{u-F}{T}} = \lambda(u)\, e^{S} .$$

$$\therefore \quad e^{-F/T} = \int \lambda(E)\, e^{S - E/T} dE = \lambda(u) \int e^{S - E/T} dE$$

(since λ varies slowly, + integrand vanishes except for $E \sim$

$$\lambda(u) = \frac{e^{-F/T}}{\int e^{S - E/T} dE}$$

$$= \frac{1}{\int e^{\frac{F-E}{T} + S} dE}$$

but $F = u - TS$

$$\therefore \lambda(u) = \frac{1}{\int e^{\frac{u - TS^{(u)} - E}{T} + S(E}\, dE}$$

$$= \frac{1}{\int e^{\frac{u-E}{T} + S(E) - S(u)}\, dE}$$

~~but~~ $S(E) - S(u) = (E-u)\frac{dS}{du} + \frac{(E-u)^2}{2}\frac{d^2S}{du^2}$

but $\frac{dS}{du} = \frac{1}{T}$;

$$\therefore \lambda(u) = \frac{1}{\int e^{\frac{(E-u)^2}{2}\frac{d^2S}{du^2}}\, dE}.$$

$$\frac{d^2S}{du^2} = \frac{d\frac{1}{T}}{du} = -\frac{1}{T^2}\frac{dT}{du}.$$

$$\therefore \lambda(u) = \frac{1}{\int e^{-\frac{(E-u)^2}{2T^2}\frac{dT}{du}}\, dE}$$

~~$= \frac{1}{\sqrt{\pi \frac{dT}{T^2 du}}}$~~

$$\frac{1}{\sqrt{\pi \frac{2T^2}{\frac{dT}{du}}}} = \sqrt{\frac{\frac{dT}{du}}{2\pi T^2}}$$

$$\therefore \rho(u) = \frac{1}{T}\sqrt{\frac{1}{2\pi}\frac{dT}{du}}\; e^{S}.$$

$$S(T) = S(0) + \left(\frac{dS}{dT}\right)_0 T + \cdots$$

but $S(0) = 0$;

$\therefore S(T) = aT + \cdots$ near zero.

also since $\frac{dF}{dT} = -S$,

$$F = -\frac{a}{2}T^2 + \cdots$$

$$\therefore U = F + TS = -\frac{a}{2}T^2 + aT^2 = \frac{a}{2}T^2 + \cdots$$

$$\therefore \rho(u) \cong \frac{1}{T}\sqrt{\frac{1}{2\pi}\cdot\frac{1}{aT}}\; e^{aT}$$

$$\cong \frac{1}{\sqrt{2\pi a}}\, T^{-3/2} e^{aT}.$$

but $T = \sqrt{\frac{2U}{a}}$

$$\therefore \rho(U) \sim \frac{1}{\sqrt{2\pi a}}\left(\frac{2U}{a}\right)^{-3/4} e^{\sqrt{2Ua}}$$

$$\therefore \rho \sim a^{\sqrt{U}}$$

Can find a by measuring level density;

4/24/40

$$\rho(u) = \frac{1}{T\sqrt{2\pi\frac{\partial U}{\partial T}}}\cdot e^{S}$$

$$= \frac{1}{T\sqrt{2\pi a T}}\, e^{aT}$$

$$\text{or } \rho(u) = \frac{1}{\left(\frac{2U}{a}\right)^{3/4}\sqrt{2\pi a}}\, e^{\sqrt{2aU}}$$

$$= \frac{a^{1/4}}{2^{5/4}\sqrt{\pi}\, U^{3/4}}\, e^{\sqrt{2aU}}.$$

$F = -\frac{a}{2}T^2 \cdots$

$S = -\frac{\partial F}{\partial T} = +aT$

$U = F + TS = \frac{a}{2}T^2$

$\frac{\partial U}{\partial T} = aT$

From γ-ray spectra & neutron ~~[illegible]~~ can say something about level densities:

For ~~[illegible]~~ $\rho \approx \frac{1}{10^5}$ (volts)$^{-1}$

$U_0 = 10^5$

$U_1 = 8 \cdot 10^6 \quad \rho \approx 1 \; (eV)^{-1}$

$$\log \frac{\rho(U_1)}{\rho(U_0)} = \sqrt{2a(U_1 - U_0)} \sim \sqrt{2aU_1} \sim 10^3 \sqrt{16a}$$

$$\log 10^5 \sim 4 \cdot 10^3 \sqrt{a}$$

$$121 \sim 16 \cdot 10^6 a$$

$$a = \frac{121}{16 \cdot 10^6} \sim 7 \cdot 10^{-6}.$$

$$T_1 = \sqrt{\frac{2U_1}{a}} \sim \sqrt{\frac{16 \cdot 10^6}{7 \cdot 10^{-6}}} = \frac{4}{3} \cdot 10^6$$

(fairly heavy nucleus) ~~T almost independent of a~~; change ρ by factor of 10 ...
T change by about 40% —

---

a —
↑↓
b —

$g_a P_{ab} = g_b P_{ba}$

g_a: statistical wt. of state; P_{ab}: probability of process a → b; P_{ba}: b → a

dE a 1st state: excited comp. nucleus
— b 2nd state: nucleus in ground state, and free particle (neutron)

$P_{ab} = \gamma_n / \hbar \qquad g_a = \rho(E)\,dE$

P_{ba} — in terms of cross section

suppose particle density in box Ω is $N(E)\,dE$

then ~~[illegible]~~ = no. of processes $= N(E)\,dE \; P_{ba}$; or

or ~~no. of~~ ~~cross-section~~ =

prob. of process = density · velocity · cross-section.

$$2 = \frac{N}{\Omega} v \sigma$$

5/1/40

$$\frac{\rho(u_1)}{\rho(u_2)} = e^{\sqrt{2cu}} = 10^5 \quad \text{at 8 meV}$$

$$\therefore e^{\sqrt{16c}} = 10^5 \qquad \therefore c = 9 \quad \text{approx.}$$

$$\frac{\rho(16)}{\rho(8)} = e^{\sqrt{18(16-8)}} = e^{3.4(\sqrt{2}-1)} = e^{12 \cdot .4} \sim 100 .$$

u	T	$\frac{1}{\rho}$
0	0	10^{-1}
8	1.3	10^{-6}
16	1.8	10^{-8}

all in m.e.v.

$$T = \sqrt{\frac{2u}{c}}$$

Principle of ~~detailed~~ balancing requires ~~that~~:

$$g_c W^c_{AP} = g_P g_A W^{AP}_c$$

C - compound nucleus
A - resulting nucleus
P - particle emitted

g's - statistical wts.

W's - probabilities of process wh

$$W^c_{AP} = \frac{\gamma^c_{AP}}{\hbar}$$

$$W^{AP}_c = \frac{v \sigma^{AP}_c}{\Omega}$$

$$g_c = \rho(u)\,du$$

$$g_A = 1$$

$$g_P = \frac{\Omega k^2 dk}{2\pi^2}$$

$$\left(\frac{\Omega 4\pi p^2 dp}{h^3} \; ; \; p = \hbar k \; ; \; \therefore = \frac{\Omega 4\pi k^2 dk}{8\pi^3} = \right.$$

$$\therefore \rho(\mathcal{U})\frac{d\mathcal{U}}{\hbar}\gamma^{c}_{AP} = \frac{\Omega k^2 dk}{2\pi^2}\cdot\frac{v\,\sigma_c^{AP}}{\Omega}$$

$$= \frac{v\,\sigma_c^{AP} k^2 dk}{2\pi^2}$$

$$\therefore \sigma_c^{AP} = \frac{2\pi^2\gamma^c_{AP}\rho(\mathcal{U})}{v k^2\hbar}\frac{d\mathcal{U}}{dk}.$$

$$\mathcal{U} = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

$$\frac{d\mathcal{U}}{dk} = \frac{2\hbar^2 k}{2m}$$

$$\therefore \sigma_c^{AP} = \frac{2\pi^2\gamma^c_{AP}\,\rho(\mathcal{U})}{v k^2\hbar}\cdot\frac{\hbar^2 k}{m}$$

$$= \frac{2\pi^2\hbar\,\gamma^c_{AP}\,\rho(\mathcal{U})}{mv\,k}$$

$mv = p = \hbar k$

~~but $\frac{\hbar}{mv} = p$~~

$$\therefore \sigma_c^{AP} = \frac{2\pi^2}{k^2}\rho(\mathcal{U})\,\gamma^c_{AP}$$

~~With~~ spin:

$g_A = 2i+1$ (i - spin of nucleus)

$g_{c_J} = \rho_J(\mathcal{U})\,d\mathcal{U}$ - for ang. mom. J ; $\rho_J(\mathcal{U}) = \frac{2J+1}{d_J}$

d_J = dist. betw. levels

$g_P = (2s+1)\frac{\Omega k^2 dk}{2\pi^2}$

Then $\sigma_{c_J}^{AP\ell} = \frac{2\pi^2}{k^2}\frac{(2J+1)}{(2i+1)(2s+1)}\cdot\frac{\gamma^{c_J}_{AP\ell}}{d_J}$

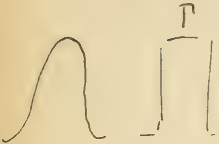
Now maximum cross section for a given ℓ is: $\sigma^{(\ell)}_{max} = \frac{\pi}{k^2}(2\ell+1)$

$\therefore \frac{\gamma}{d} \sim 1$ small wave length: $\sum_0^{kR}\sigma^{(\ell)}_{max} = \frac{\pi}{k^2}\sum_0^{kR}(2\ell+1) = \pi R^2$

5/8/40

$$\int \sigma_{cJ}^{AP}\, dE = \frac{2\pi^2}{k^2}\, \frac{2J+1}{(2i+1)(2s+1)}\, \gamma_{cJ}^{AP}$$

$$\left(\sigma_{cJ}^{AP}\right)_{Max} \sim \frac{2\pi^2}{k^2}\, \frac{\gamma_{cJ}^{AP}}{\Gamma_{cJ}}.$$



$$\left(\sigma^{AP \to cJ}_{\ \ \swarrow BQ}\right)_{max} = \left(\sigma_{cJ}^{AP}\right)_{Max} \frac{\gamma_{BQ}^{cJ}}{\Gamma_{cJ}} = \frac{2\pi^2}{k_P^2}\, \frac{\gamma_{AP}^{cJ}\, \gamma_{BQ}^{cJ}}{(\Gamma_{cJ})^2}.$$

Bethe & Placzek: treat comp. nucleus like photon in Dirac radiation theory — ~~introduce~~ ~~interaction~~ w. particles

Kapur, Peierls; ~~investigate~~ change in boundary conditions —

First: ⓒ χ_c; ~~[illegible]~~ Then Ⓑ $\psi_B \cdot \varphi_P^{OP}$.

Wigner regards $\chi_c = 0$ for $r > r_0$;

but this would need infinite ~~potential~~

so ~~this~~ ~~finite~~ ~~potential~~:

$$\frac{d^2\varphi}{dr^2} + \frac{2m}{\hbar^2}(E-U) = 0 \quad r < r_0$$

$$\frac{d^2\varphi}{dr^2} + k^2\varphi = 0 \quad r \geq 0 \quad k^2 = \frac{2mE}{\hbar^2}$$

$$\varphi = I\,\frac{\sin kr}{k} + S e^{ikr} \qquad \varphi' = I\cos kr + ikS e^{ikr}$$

$\therefore ik\varphi - \varphi' = iI\sin kr - I\cos kr = I e^{-ikr}$

If system can only decay: only outgoing waves;

no incoming wave if: $\frac{d\varphi}{dr} = ik\varphi$; (then $I = 0$).

can satisfy this & 1st eq'n only for certain values of k - (complex)

of $W_n = E_n + i\frac{\gamma_n}{2}$.

$$e^{\frac{iW_n t}{\hbar}} = e^{\frac{iE_n t}{\hbar}} e^{-\frac{\gamma_n t}{\hbar}}$$

∴ complex eigen value means exponential decay of nucleus;

5/15/40

~~γ_n~~ =

5/15/40

$$\gamma_n = \frac{\hbar^2}{2m} |\varphi_n(r_0)|^2 k_n \qquad W_n = E_n - \frac{i\gamma_n}{2}$$

$$k_n = \frac{\sqrt{2mE_n}}{\hbar}$$

I incident wave present: $\qquad k$ – wave vector of outgoing wave

boundary cond.; $I e^{-ikr} = \left(\frac{d\varphi}{dr}\right)_{r_0} - ik\varphi(r_0)$

solutions φ_n (of eqn without incident wave)

build up $\varphi = \sum a_n \varphi_n + \chi$, which satisfies boundary condition – (incident wave)

then $\left(\frac{d\chi}{dr}\right)_{r_0} - ik\chi(r_0) = I e^{-ikr_0}$

$\left(\varphi = I \frac{\sin kr}{kr} + S e^{ik\ldots}\right.$

can determine a_n's ;

Kapur, Peierls
Proc. Roy. Soc. 166

$$S = \cos kr_0 \, \varphi(r_0) - \frac{1}{k}\sin(kr_0)\left(\frac{d\varphi}{dr}\right)_{r_0}$$

$$\sigma_B^A = \frac{4\pi |S|^2}{|I|^2} = \frac{\pi}{k^2}\left|\sum_n \frac{u_A^n u_B^{n*}}{(W_n - E)N_n}\right|^2 \qquad E: \text{energy, incident p}\ldots$$

$\gamma_A^n = |u_A^n|^2$

$N_n = \int_{\text{nucleus}} \varphi_n^2 \, d\tau$

$= 1$ almost

When level density is great, can separate nuclear quantities ~~into~~ those varying slowly (order of nucl[ear] MeV binding energies) & those " fast (order of level separation ~~β~~ volts). Here only slow variations important; if $E < |W_n|$ $\qquad$ $(E \ll E_n,\ E \ll \gamma_n$

$|W_n - E| \gg \Delta E$

σ varies only as $1/k^2$

reaction probability $= P = j\sigma = \rho P$) but $j = \rho v$, $\therefore \sigma = \frac{P}{v}$

5/17/40

$$\sigma_B^A = \frac{\pi}{k^2}\left|\sum_n \frac{u_A^n u_B^n}{(W_n - E)}\right| \qquad |u_A^n|^2 = \gamma_A^n$$

$$W_n = E_n - i\frac{\gamma_n}{2}$$

If near resonance, so that only one term ~~contributes~~ to cross-section:

$$\sigma_B^A\rangle_n = \frac{\pi}{k^2}\frac{\gamma_A^n \gamma_B^n}{(E_n - E)^2 + (\gamma_{n/2})^2}$$

or, if $x = \dfrac{E - E_n}{\gamma_n/2}$

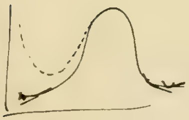
$$\sigma_B^A\rangle_n = \frac{4\pi}{k^2}\frac{\gamma_A^n\gamma_B^n}{\gamma_n^2}\frac{1}{1+x^2} = \frac{\sigma_{0\,B}^{A,n}}{1+x^2}$$

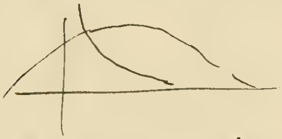
$$\int \sigma_B^{A,n}\, dE = \gamma_{n/2}\int \frac{\sigma_{0\,B}^{A,n}}{1+x^2}\,dx = \sigma_{0\,B}^{A,n}\cdot\frac{\gamma_n}{2}\cdot\pi$$

$$\therefore = \frac{2\pi^2}{k^2}\frac{\gamma_A^n\gamma_B^n}{\gamma_n} \quad ;$$

comparing with 5/8/40, we see that, if there are angular momenta, statistical factor $\dfrac{(2J+1)}{(2i+1)(2s+1)}$ is added:

$$\therefore \sigma_B^{A,n} = \frac{4\pi}{k^2}\frac{(2J+1)}{(2i+1)(2s+1)}\frac{\gamma_A^n\gamma_B^n}{(\gamma^n)^2}\frac{1}{1+x^2}$$





$$\sigma_0 = \sim \frac{\Gamma_A^n\Gamma_B^n}{(\Gamma^n)^2}$$

$$\Gamma_A^n = \frac{k_n}{k_0}\gamma_A^n \; : \quad \sigma_B^{A,n} = \sigma_{0\,B}^{A,n}\cdot\frac{1}{1+x^2}\cdot\frac{k_0}{k}$$

because for slow particles, γ_B^n is practically indep. of k —

$$J = \begin{matrix} i+s \\ \vdots \\ |i-s| \end{matrix}$$

if a neutron: $J = i \pm \frac{1}{2}$ $\quad\therefore \dfrac{2J+1}{(2i+1)\cdot 2} = \dfrac{2i \pm 1 + 1}{(2i+1)2} = \dfrac{1}{2} \pm \dfrac{1}{2(2i+1)}$

§ low neutrons:

~~Radiate~~

Total cross section : $\sigma_{max} = \frac{2\pi}{k^2}\left\{1 \pm \frac{1}{2i+1}\right\}\frac{\Gamma_{neutron}}{\Gamma_r}$ $\left(\frac{\Gamma_1}{\Gamma_1+\Gamma_{ne}} \sim 1\right)$

Γ_{ne} - neutron width. $\Gamma_n \sim \Gamma_1$ radiation width ~ (total due to particles)

$$\sigma_{scattering} = \sigma_{max}\left(\frac{\Gamma_{ne}}{\Gamma_1}\right) = \frac{2\pi}{k^2}\left(1 \pm \frac{1}{2i+1}\right)\left(\frac{\Gamma_{ne}}{\Gamma_r}\right)^2$$

5/22/40

$j = \rho v$ $\qquad j\sigma = \rho p$

$\therefore \sigma = \frac{p}{v}$

in system moving with velocity v':

$$\sigma(v) = \frac{v'}{v}\sigma(v')$$

---

v - vel. in observer's system.

v' - vel. in system in which nucleus at rest

u - vel. of nucleus.

then $v = v' + u_x$

$v' = v - u_x$

$E' - E = m v u_x = \sqrt{2mE}\, u_x$

If Maxwell distribution : $w(u_x)du_x = \left(\frac{M}{2\pi kT}\right)^{1/2} e^{-\frac{Mu_x^2}{2kT}} du_x$

$$\sigma(E) = \int \sigma(E')\, W(E')\, dE'$$

We had: $\sigma(E') = \dfrac{\sigma_0}{1+\left(\frac{E'-E_0}{\Gamma/2}\right)^2}$ put: $\dfrac{E'-E_0}{\Gamma/2} = x'$

$\dfrac{E-E_0}{\Gamma/2} = x$

$\therefore \sigma(E) = \sigma_0 \int \dfrac{}{1+x'^2}$

$w(E')\,dE' = \left(\dfrac{M}{2\pi kT}\right)^{1/2} \dfrac{1}{\sqrt{2mE}} e^{-\frac{M}{2kTm}\cdots}$

$\therefore \sigma(E) = \dfrac{\sigma_0 \quad \Gamma/2}{\sqrt{4\pi kTE\frac{m}{M}}} \int \dfrac{e^{-\frac{M(x-x')^2\,\Gamma/2}{4kTmE_0}}\,dx'}{1+x'^2}.$

call: $4\sqrt{kTE_0\frac{m}{M}} = \Delta$ $x'-x = \sqrt{2mE_0}\cdots$

$\Gamma/\Delta = \xi.$ $\Gamma/2$

$\sigma(E) = \sigma_0(\pi)^{-1/2}\xi \int \dfrac{e^{-\xi^2(x-x')^2}}{1+x'^2}\,dx'$

if ξ is large, almost whole integral comes from near $x'=x$;

$\therefore \sigma = \dfrac{\sigma_0\,\xi}{\sqrt{\pi}(1+x^2)} \cdot \dfrac{\sqrt{\pi}}{\xi} = \dfrac{\sigma_0}{1+x^2}.$

if ξ is small, $\sigma = \dfrac{\sigma_0}{\sqrt{\pi}}\xi\, e^{-\xi^2x^2} \int \dfrac{dx'}{1+x'^2}$

$= \sigma_0\sqrt{\pi}\,\xi\, e^{-\xi^2x^2}$

$\xi x = \dfrac{E-E_0}{\Delta/2}$

$= \sigma_0\sqrt{\pi}\dfrac{\Gamma}{\Delta}\, e^{-\left(\frac{E-E_0}{\Delta/2}\right)^2}$

really doppler effect – Δ – doppler width.

$\Delta = 4\sqrt{h T \frac{m}{M} E_0} \sim \frac{4}{\sqrt{40}} \frac{1}{\sqrt{100}} \cdot \sqrt{E} \sim \frac{1}{15}\sqrt{E}$ –

This was all for gases –

for solids : same results, if $W \gg k\Theta$ – only –
substitute mean energy, degree of freedom for temperature –

5/24/40

$\xi \gg 1 : \sigma = \frac{\sigma_0}{1+x^2}$; $\int \sigma \, dE = \pi \frac{\Gamma}{2} \sigma_0$

$\xi \ll 1 \quad \sigma = \sigma_0 \xi \sqrt{\pi} \, e^{-\xi^2 x^2}$ – Doppler

---

neutron, energy E_0 passes into water or paraffin; what will be energy distribution ? –

Q neutrons per sec.

(logarithmic count)

$$\rho(E) \frac{dE}{E} = Q \frac{l(v)}{v} K(x) \frac{dx}{x} \qquad x = \frac{E_0}{E}.$$

$l(v)$: mean free path; $\frac{l(v)}{v}$. lifetime

($K(x)\,dx$ represents av. no of collisions in dx)

$k(x,x')\frac{dx}{x}$: energy distribution after collision –

$\therefore$ ~~total~~ no. going from x to x' is : $\int_1^x k(x,x') K(x') \frac{dx\,dx'}{x\,x'}$

$$\therefore \quad K(x) = k(x,1) + \int_1^x k(x,x') K(x') \frac{dx'}{x'}$$

For neutron - proton collision, $k(x,x')\frac{dx}{x} = \frac{dE}{E'} = -\frac{x'}{x}\frac{dx}{x}$

$$\therefore \quad K(x) = \frac{1}{x} + \int_1^x \frac{x'}{x} K(x') \frac{dx'}{x'} ; \quad K(x) = \frac{1}{x}\left[1 + \int_1^x K(x')\,dx'\right]$$

$$\therefore K(x) = 1$$

$$\therefore \quad \rho(E)\frac{dE}{E} = \frac{Q\,\ell(v)}{v}\frac{dE}{E} = 2Q\,\ell(v)\frac{dv}{v^2}$$

$$B^{10} + n = Li^7 + He^4$$

9/29/40

β – disintegration

(The essential differen between β^+ [illegible]

We found: if $Z^A > (Z+1)^A$, get $Z^A \to (Z+1)^A + \beta$
(atomic wts.)

if $(Z+1)^A > Z^A + 2m$: get $(Z+1)^A \to Z^A + \beta$

[illegible]

[illegible] ν are unknown — call it [illegible]

$P \leftarrow N + \beta + \nu$

if proton goes to left, positron annihilated [illegible]

[illegible] electron [illegible]

annihilation of neutrino is same as production of anti [illegible]

i.e. ; $N \rightleftarrows P + \beta^- + \nu'$

$N - (P + \beta^-) = .75$ Mev

The possible energetically —

have .75 Mev to distribute

[illegible]

[illegible]

[illegible]

[illegible] H [illegible]

[illegible]

$= 2.2$ [illegible]

[illegible] $\frac{1}{m}$ [illegible]

[illegible] H [illegible]

[illegible] $= 2.2$ [illegible]

[illegible]

[illegible] 1.1 [illegible]

[illegible] $= 1.1$ [illegible]

[illegible]

[illegible]

[illegible]

[illegible] H [illegible]

[illegible]

[illegible]

$H =$ [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

(most energetic [illegible]) [illegible]

∴ take $\varphi_n = \varphi_{el} = 1$

then we have : $|a_n^+ a_{el}|^2 \mathcal{G}^2$, where $\mathcal{G} = \int d\tau \, u_n^+ u_p$

⟶ averaged over space region gives 1.

$\bar{\mathcal{G}} \cong 1$

$\varepsilon = \frac{E_e}{mc^2}$

$\varepsilon_0 = \frac{E_{max}}{mc^2}$

$$\sim \int \sqrt{\varepsilon^2 - 1}\, \varepsilon (\varepsilon_0 - \varepsilon)^2 \, d\varepsilon$$

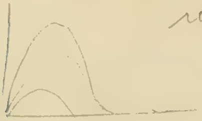
max. at : [illegible]

Konopinski & Uhlenbeck ; additional ...

in M ; hence $\alpha(E)$ has additional $(E_0-E)^2$ ...

... than suggested ...

washed well with ...



... ; can find $\frac{ft}{|M|^2}$; largest values ...

... ; largest for

mirror nuclei H^3–He^3, Li^7–Be^7

Natural radioactive nuclei $|M|^2 \sim \frac{1}{100}$

[illegible] $\Delta J = 0$ [illegible]

[illegible]

Gamow-Teller. [illegible] $\Delta J = 1, 0$

[illegible] $\Delta J > 1$ [illegible]

[illegible]

[illegible]

[illegible]

(Allowed) $\Delta J =$ 1

Forbidden $\Delta J = 1$ $10^{[illegible]}$

$\Delta J = 2$ $10^{[illegible]}$

[illegible]

$n_a = n_b$.

get $\Delta j = \pm 1$, mean el. moment $= \int u_a^* P_a^j \sum e\xi_i \, u_a P_a^j$

$j \to j-1$: energy change $= \frac{\hbar^2}{2M'R_0^2}\left[j(j+1) - (j-1)j\right] = \frac{\hbar^2}{I_0} j$.

[illegible]

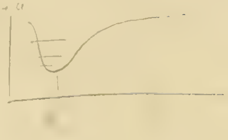
rotation of molecule —

$$[illegible] \left\{ [illegible] + \frac{h^2}{2M'}\frac{a_1}{R^2} + W_1(R) + \bar{U}_e(R) \right\} P = W P$$

vibration of nuclei —

$\frac{X^2}{2M}\frac{a_1}{R^2} + W + \bar{U}$

curve of potential energy



[illegible]

$$W = W_e(R_0) + \bar{U}_e(R_0) + \frac{X^2}{2M'}\frac{a_1}{R_0^2} + W_v$$

[illegible]

$$[illegible] + W_e(R) - W_e(R_0) + \bar{U}_e(R) - \bar{U}_e(R_0) = W_v P$$

[illegible]

[illegible]

$\Psi = e^{\,[illegible]}\,\Delta(\theta)\,[illegible]$

$-\frac{\hbar^2}{2M}\Big[\,\Delta\big(e^{[illegible]}\,\Theta F\big) - [illegible]\,\frac{\partial}{\partial R} [illegible] \frac{\partial^2}{\partial R^2}$

$+ \frac{F}{R^2}\Big[-i\,[illegible]\;\frac{[illegible]}{\sin\theta}\,[illegible]$

$-L_{x'}^2 - L_{y'}^2 - L_{z'}^2 \cot^2\theta + (L_{x'}L_{z'} + L_{z'}L_{x'})\cot\theta \Big] u \Big\}\Big]$

$+\, e^{[illegible]}\,\Theta F \Big[\, [illegible] - \sum [illegible] \Big]$

[illegible]

Take first [illegible]

[illegible]

[illegible]

$z = R \cos\Theta$

$x_T' = x_T \cos\phi \cos\Theta + y_T \sin\phi \cos\Theta - z_T \sin\Theta$

$y_T' = -x_T \sin\phi + y_T \cos\phi$

$z_T' = x_T \cos\phi \sin\Theta + y_T \sin\phi \sin\Theta + z_T \cos\Theta$

$$-\frac{\hbar^2}{2M'}\left\{\Delta + \frac{1}{R^2}\left[-i\left(\cot\Theta + \frac{\partial}{\partial\Theta}\right)L_\eta' + \frac{2i}{\sin\Theta}\left(L_\xi' - L_\zeta' \cot\Theta\right)\frac{\partial}{\partial\phi} \ldots\right]\right.$$

$$+ \frac{Z_1 Z_2 e^2}{R} - \sum e^2\left(\frac{Z_1}{r_{1i}} + \frac{Z_2}{r_{2i}}\right) + \sum\sum \frac{e^2}{r_{ij}} - \frac{\hbar^2}{2m} \ldots$$

or $H_I = $ [illegible]

F [illegible] H_1 [illegible]

$W_1 = A[$[illegible]$]$

$\delta W_1 = $ [illegible]

[illegible]

Band [illegible] [illegible]

[illegible]

$H = $ [illegible]

$+$ [illegible]

[illegible]

[illegible]

[illegible] [illegible] 1. [illegible]

[illegible]

[illegible]

$\Delta g = $ [illegible] $\Delta M = $ [illegible]

[illegible]

$J + I = F$, total total ang. mom.

Show ψ's to diagonaliz.

Hamiltonian [illegible]

$H_0 u(j) = W_j^0 u(j) \quad \frac{\partial H}{\partial t} = -\frac{i}{\hbar}$

Then $\dot{a}_j = -i \zeta \sum_k a_k H_{jk} e^{\ldots}$

If $t = 0$ let $\psi = u_n$. Then approx. $\dot{a}_j = -i \zeta H_{jn} e^{\ldots}$

$a_j = -H_{jn}$

$= P_z - L_z + L_{z+1} - L'_z$

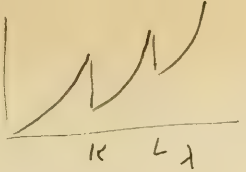
K_α line $\rightarrow 0$ p...

or: $(KL)_z - (LM)_z = K_z - L_z + (L_{z+1} - M_{z+1})$

K_β

(KL) (LL)

$3P$

Absorption : as. 

(at discontinuities, wave length is just too high (i.e. energy just too low) to ~~free the~~ eject K, L, electrons respectively).

Use H energies:

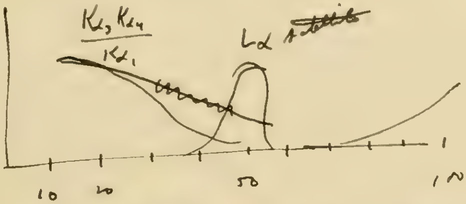
$$W = \frac{1}{\alpha^2}\left[\frac{1}{\sqrt{1+\frac{\alpha^2 Z^2}{(n-j-\frac{1}{2}+\sqrt{(j+\frac{1}{2})^2-\alpha^2 Z^2})^2}}} - 1\right]$$

$$= -\frac{Z^2}{2n^2} - \frac{\alpha^2 Z^4}{2n^3}\left(\frac{2}{2j+1} - \frac{3}{4}\frac{1}{n}\right).$$

To fit K shells of heavier atoms, must use a screening constant : $(Z-\sigma)$ for Z.

Other lines are observed as well – (usually on short wave length side –

"Satellites":



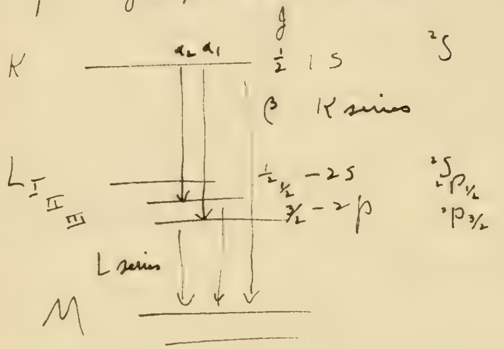
Wentzel assumed ~~satellites~~ due to ionized atoms – but (e.g. both K electrons might be gone, then KK → KL) but excitation ~~of~~ energy wasn't twice the usual (as it should be on that theory)

X-rays –

X-ray spectra: remove interior electrons:

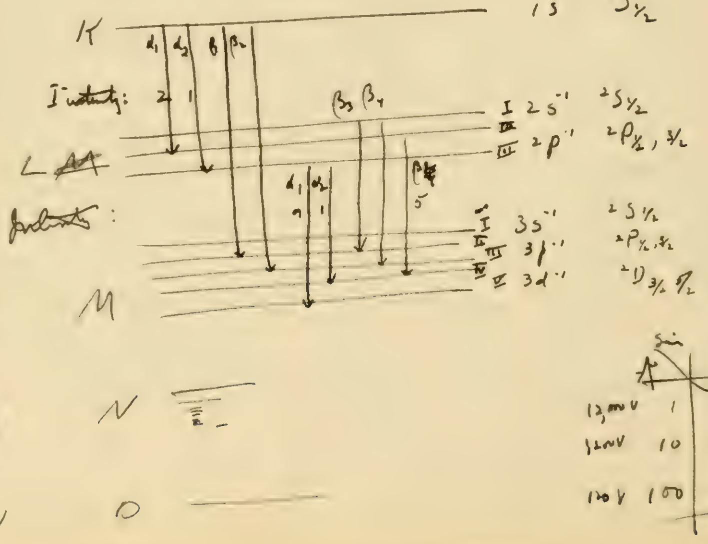
$K:2:1s^2$
$L:8:2s^2\,2p^6$
$M:18:3s^2\,3p^6\,3d^{10}$
N: ...
O:

Suppose a K electron missing: high frequency lines from jumps to K shell.



α_1 brighter

5/23/40 | Cadmium: $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 5s^2$ (K · L; M; N; O) 48



Left margin: 3 000; 445; 390; 86; 35; 1.3; 1

		K	L	M	N
13,000 V	1	33	74–50	—	[illegible]
12000 V	10	12	32	55–45	[illegible]
120 V	100	5	15	30	70–

If $S = 0$: $g = 1$ Lorentz case.

$L = 0$, $g = 2$

Pattern same for a given series, since L, S remain the same, hence g the same for all –

(Largest g : $2\frac{2}{3}$;
smallest negative.

C. Strong Field : $\frac{e\hbar}{2mc} \sum (L_{z_j} + 2S_{z_j}) F$ comparable to spin orbit interaction.

Incomplete Paschen-Back Effect:

Field effect >> spin orbit in L-S coupling
" >> elect. inter. in j·j "

L-S coupling : L, S quantum no's persist; but g^2 quantization is spoiled – can use $L\,S\,\lambda\,\Sigma$

$$W_F = \frac{|e|\hbar}{2mc}(M + \Sigma)F. \qquad \Delta M = 0, \pm 1 \quad \delta\Sigma = 0.$$

j·j coupling –

D Strong field : >> everything else.

Start with $\prod_{j=1}^{N} n_j l_j \lambda_j \sigma_j$

then $W_F = \frac{|e|\hbar}{2mc} F (\underbrace{\sum \lambda_0 + 2\sum \sigma}_{M + \Sigma})$

Can have L, S quantum no's.

5/21/40 A. In general $\mathcal{J}_3$ can be diagonal: $1 - M$.

$$\Delta M = 0, \pm 1 .$$

$$\pi \quad \sigma$$

B. Weak Field case. Then have $\mathcal{J}, M$.

Now $[\mathcal{J}, (L_3 + 2S_3)] = -i\, \mathcal{J} \times (L_3 + 2S_3)$

$\therefore W_F : g \frac{|e|\hbar}{2mc} F M$ g a const., Landé factor

line shift: $h\Delta\nu = \frac{|e|\hbar}{2mc} F (g'M' - g''M'')$

$\frac{|e|\hbar}{2mc} F [(g' - g'')M' - g''(\Delta M)]$ $\Delta M = M'' - M'$

if $g' = g'' = g$: $h\Delta\nu = \frac{|e|\hbar}{2mc} g \Delta M$; $\Delta M = 0, \pm 1$,

Triplet.

If we have L-S coupling: $L\, S\, \mathcal{J}\, M$

$$\overline{L_3} = \frac{\mathcal{J}(\mathcal{J}+1) + L(L+1) - S(S+1)}{2\mathcal{J}(\mathcal{J}+1)} M$$

Zeeman ~~perturbation~~: $-\frac{e\hbar}{2mc} \sum (L_{r3} + 2S_{r3}) F$.

$$L_3 + 2S_3 = 2\mathcal{J}_3 - L_3 =$$

$$2M - \overline{L_3} =$$

$$\therefore g = \frac{2\overline{\mathcal{J}_3} - \overline{L_3}}{M} = \frac{~~2M - \mathcal{J}(\mathcal{J}+1)~~ \quad 2M - \frac{\mathcal{J}(\mathcal{J}+1) + L(L+1) - S(S+1)}{2\mathcal{J}(\mathcal{J}+1)}}{M}$$

$$= 2 - \frac{1}{2} - \frac{L(L+1) - S(S+1)}{2\mathcal{J}(\mathcal{J}+1)}$$

$$= \frac{3}{2} + \frac{S(S+1) - L(L+1)}{2\mathcal{J}(\mathcal{J}+1)}$$

$$Q_\varepsilon = \sum_{\rho<\tau=1}^{\varepsilon} \left[J_{\tau\rho} - K_{\tau\rho}\, \delta^{\sigma_\tau}_{\sigma_\rho} \right]$$ (same spins).

$$= \frac{1}{2} \sum_{\rho=1}^{\varepsilon} \sum_{\tau=1}^{\varepsilon} \left[J_{\tau\rho} - K_{\tau\rho}\, \delta^{\sigma_\tau}_{\sigma_\rho} \right]$$

Now associate with ψ that ψ' obtained by ... :

For it : $Q_{m-\varepsilon} = \frac{1}{2} \sum_{\rho=\varepsilon+1}^{m} \sum_{\tau=\varepsilon+1}^{m} \left(J_{\tau\rho} - K_{\tau\rho}\, \delta^{\sigma_\tau}_{\sigma_\rho} \right)$ ... [illegible]

$m = 2(l+1)$

$$Q_\varepsilon + \frac{1}{2} \sum_{\rho=1}^{\varepsilon} \sum_{\tau=\varepsilon+1}^{m} \left[\quad \right] = \frac{1}{2} \sum_{\rho=1}^{\varepsilon} \sum_{\tau=1}^{m} \left[J_{\tau\rho} - K_{\tau\rho}\, \delta^{\sigma_\tau}_{\sigma_\rho} \right]$$

B, indep. of l, σ

$$\therefore Q_\varepsilon = \tfrac{1}{2} \varepsilon B,$$

similarly : $Q_{m-\varepsilon} = \frac{1}{2}(m-\varepsilon)B$

$$\therefore Q_{m-\varepsilon} - Q_\varepsilon = \left(\tfrac{1}{2} m - \varepsilon\right) B$$ ... elst. energies

for ψ + associated ψ' differ only by a constant ; ∴ same term spacing.

Spacing:

1st order multiplet spacing in LS coupling is just reversed [illegible] usually

Larger at less below

This given by diagonal elements in spin-orbit interaction, using LS or JM.

Spin-orbit Matrix in terms of corresp. ψ's = − ([illegible] ψ.)

Radial integrals identical throughout, (depend only on n l) . consider only angular part.

Diagonal elements: $\propto \sum \lambda_\tau \sigma_\tau$ for ε electrons.

But $\sum_{\varepsilon'} \lambda_\tau \sigma_\tau = \sum_{\text{(whole shell)}} \lambda_\tau \sigma_\tau - \sum_{\text{omitted}} \lambda_\tau \sigma_\tau = - \sum_{\varepsilon} \lambda_\tau \sigma_\tau$,

since $\sum_{\text{whole shell}} \lambda \sigma = 0$.

Non-diagonal elements: occur in product ψ's differing in only 1 one-electron ψ's (1st order), & then only when λ differs by 1 —

Can show here also just a reversal of sign.

Spacing of terms (1st order) same for ε present, ε absent, i.e. except [illegible] triplet (at least if only [illegible] the third [illegible] in the configuration) — then [illegible]

Prove diag. elements of elect. inter. is same for ψ & ψ'.

"Holes" in subshells - behave somewhat like single electrons (sometimes signs reversed, etc.).

L-S spacing the same; multiplet spacing reversed - bigger J's below -

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no. of el. $\div 2(2l+1)$ taken ε at a time

5/18/40 (Sam's notes →)

1. ε electrons outside - + closed subshells.

2. ε electrons missing: $2(2\lambda+1) - \varepsilon = \varepsilon'$, vacant

Same no. of wave f'ns for ε or ε' $= {}^{2(2l+1)}C_{\varepsilon}$ ${}^{2(2l+1)}C_{\varepsilon'}$

We set up a correspondence ψ for e electrons $\lambda_1 \sigma_1) \cdots (\lambda_\varepsilon \sigma_\varepsilon)$

ψ' " e' omit $(-\lambda_1 -\sigma_1) \cdots (-\lambda_\varepsilon -\sigma_\varepsilon)$

e.g. $l:1$: $1\alpha \quad 0\alpha \quad -1\alpha \quad 1\beta \quad 0\beta \quad -1\beta$

<— present —>

$\varepsilon = 2$ $\psi_{1\alpha\, 0\beta}$

$\varepsilon' = 4$ × ← missing

$\Lambda = \sum \lambda$

(since $\Lambda + \Sigma$ for closed shell are zero and $0 - \Sigma - \lambda_i = \Sigma\lambda_i$) same for both; $\therefore$ $LS\Lambda\Sigma$ or $LSJM$ f'ns as Σ ...

Same lin. comb. of ψ + ψ'

$\therefore$ same ~~terms~~

e.g.: $L_p(\lambda_1\sigma_1) = K_1^{\lambda_1}(\lambda_1+1, \sigma_1)$ if (λ_1+1, σ_1) is missing in ψ

Then $(-\lambda_1, -\sigma_1)$ is absent in ψ', $-(\lambda_1+1), \sigma_1)$ is present

ψ' ... $-p[-(\lambda_1+1), \sigma_1]$ $K_1^{-\lambda_1+1}$ (λ_1, σ_1)

... $K_1^{\lambda_1}$ $(-\lambda_1, -\sigma_1)$ ... same form.

1st order energies; Spin-orbit energy = 0 (since it depends on $\Sigma \lambda\sigma$, which = 0 here)

Elst. $Q = \sum_{\tau=1}^{2l+1} J_{\tau\tau} + 2 \sum_{\tau<\rho=1}^{2l+1} (2J_{\tau\rho} - K_{\tau\rho})$.

$$J_{\tau\rho} = \int \psi_\tau^*(q_1)\psi_\rho^*(q_2)\frac{e^2}{r_{12}}\psi_\tau(q_1)\psi_\rho(q_2)$$

K

Closed shells + ε electrons in a not closed config: get same ~~multiplicity~~ as if subshells not there; also same intervals, (grounds so long as no change occurs in subshells

Spin orbit energy = that of the ε electrons –

Elst energy: (a) subshell interaction: shift all levels same amt.
(b) interaction of ε els., as tho they were alone
(c) " " " with subshells –

c) $\bar{Q} = \sum_{\tau=1}^{2(2l+1)} (2J_{\tau\kappa} - K_{\tau\kappa})$ for single electron $\psi_{n_\kappa l_\kappa \lambda_\kappa \sigma_\kappa}$

+ subshells with $2\nu_0$ els. independent of spin σ_κ; also indep of λ_κ

Product f'n: $\psi_{subs}\,\psi_\kappa$; Then $L_p \psi_s \psi_\kappa = h_{l\kappa}^{\lambda\kappa} \psi_s \psi_{\kappa+1}$

$$\int \psi_s^* \psi_\kappa^* \sum_{\tau=1}^{2\nu_0} \frac{e^2}{r_{\tau\kappa}} \psi_s \psi_\kappa = \frac{1}{h_{l\kappa}^{\lambda\kappa}} \int (L_d \psi_s \psi_{\kappa+1})^* (\Sigma\ )\, \psi_s \psi_\kappa .$$

$$= \int \psi_s^* \psi_{\kappa+1}^* (\Sigma \mid \psi_s \psi_{\kappa+1}$$

∴ elst. interaction of ε electrons with closed subshells shift all terms in a config the same – not affecting term spacing

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As ε → closed shell, less f'ns to choose from;

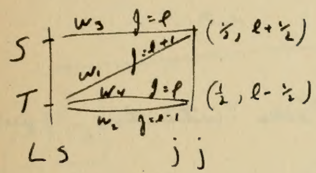
if ε = closed shell − 1, same no. of possibilities as when ε = 1

5/16/40 Intermediate coupling –

$n s\, n' p$.

$m = \lambda$:

$$\psi_1 \; (0\alpha, \lambda-1, \alpha) \quad \psi_2 \; (0\alpha, \lambda\beta) \quad \psi_3 \; (0\beta\, \lambda\alpha) \quad \psi_4 \; (0\beta\, \lambda+1, \beta)$$



$$\psi_1' = \frac{1}{D_1}\sqrt{(l+\lambda)(l+\lambda+1)}\;\psi_1 + \frac{1}{D_1}\sqrt{(l+\lambda+1)(l-\lambda+1)}\;\psi_2$$
$$+ \frac{1}{D_1}\sqrt{(l+\lambda+1)(l-\lambda+1)}\;\psi_3 + \frac{1}{D_1}\sqrt{(l-\lambda)(l-\lambda+1)}\;\psi_4$$

$$D_1 = \sqrt{2(l+1)(2l+1)}$$.

ψ_2' . . .

ψ_3'

ψ_4'

$$\Delta W_n = R_{nn} + \sum_{j \neq n} \frac{|R_{nj}|^2}{W_n - W_j}$$

always a repulsive effect;

Closed shells + subshells –

Zero order pert. Theory

Config nl $(l\sigma:)$ $l\alpha$, $l-1, \alpha$ ..... $-l, \alpha$
$l\beta$, $l-1, \beta$ ..... $-l, \beta$

~~total~~ $2(2l+1)$ el.'s closed sub-shell

For a given n: all full . closed shell .

$$\Lambda = \Sigma\lambda = 0 \; ; \quad \Sigma = \Sigma\sigma = 0$$

$$\therefore L = 0, \; S = 0, \; J = 0 \quad M = 0 \qquad {}^1S_0$$

e.g. Noble gases – lowest term always ${}^1S_0$

$$\frac{dx}{d(\frac{1}{x})} = \frac{d\,(\frac{1}{x})^{-1}}{d(\frac{1}{x})}$$

$$= -\left(\frac{1}{x}\right)^{-2}$$

$$= -x^{2}$$

$$\frac{du}{d\left(\frac{1}{x}\right)} = \frac{du}{dx} \cdot \frac{dx}{d(\frac{1}{x})} = -x^{2}\frac{du}{dx}$$

$$=$$